Notes for Lectures in Quantum Mechanics *

Approximating matter radiation interactions

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1 Charged Particle Interaction with EM Field

The expression for force on a point charge q at a position $\vec{r}$ is

$$\vec{F} = q\big(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\big) \tag{1}$$

The classical mechanics equations of motion can then be set up using Newton's second law of motion. These equations can be derived from the Lagrangian formalism if we take the Lagrangian to be

$$L = \frac{1}{2}m\vec{r}^2 + \frac{q}{c}\vec{A}\cdot\vec{r} - q\phi(\vec{r}) \tag{2}$$

The corresponding Hamiltonian is given by

$$H = \frac{1}{2m}\Big(p - \frac{e}{c}\vec{A}\Big)^2 + q\phi(\vec{r}) \tag{3}$$

Thus we can set up the Lagrangian or the Hamiltonian formalism as needed. It must be noted that the electric and magnetic fields do not enter the Lagrangian or the Hamiltonian, these contain only the potentials which are subject to gauge transformations and one must discuss the gauge invariance of the formulation.s In fact it is easy to see that the equations of motion do not change.

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2 Schrödinger Equation

The Hamiltonian operator for a charged particle in external e.m. fields is obtained from the classical Hamiltonian, Eq.(3), by the replacement

$$\vec{p} \longrightarrow \widehat{\vec{p}} = -i\hbar\nabla \tag{4}$$

Thus the Schrödinger equation takes the form

$$i\hbar\frac{d\psi}{dt} = \widehat{H}\psi \tag{5}$$

where the Hamiltonian operator is given by

$$\widehat{H} = \frac{1}{2m}\left(\widehat{\vec{p}} - \frac{e}{c}\vec{A}\right)^2 + q\phi(\vec{r}) \tag{6}$$

3 Gauge Invariance

Here again we must ask if our quantum mechanical description of the charged particle is gauge invariant? Note that the potentials $\vec{A}, \phi$, appearing in the Schrödinger equation, are subject to gauge transformations and a gauge transformation should not have any effect on the observable quantities. One can in fact prove that the description is indeed gauge invariant.

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