Notes for Lectures in Quantum Mechanics [1]

Symmetrization Postulate

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Overview

Introduction

We now introduce symmetrization postulate for a system of several identical particles. For a system of two identical particles wave function must be chosen to be symmetric or antisymmetric under exchange of space and spin variables, However, at this stage, theoretical considerations alone in non relativistic quantum mechanics, do not help us in deciding which one, symmetric or anti symmetric wave function is the correct choice for a given particle. An appeal to experiment does give an answer which is contained in the statement of the symmetrization postulate. Basically the symmetry property of the total wave function of a system of identical particles is tied to the spin of the particle.

The Symmetrization Postulate 1/2

The symmetrization postulate states that

- For a system of **two** identical particles with integral spin,known as bosons, the total wave function must be symmetric under simultaneous exchange of all the variables such as the space and spin variables. For a system of two identical particles of half odd integral spin, known as fermions,. the full wave function must be anti-symmetric under a simultaneous exchange of all the variables such as the space and spin variables.

  If ξ_1, ξ_2 denote the set of all variables such as, space and spin, of two identical particles. Then the symmetrization postulate states that the total wave function $\psi(\xi_1, \xi_2)$ must be symmetric for bosons and antisymmetric for fermions under an exchange of ξ_1 and ξ_2.

$$\psi(\xi_2, \xi_1) = +\psi(\xi_1, \xi_2) \qquad (\text{ bosons}) \qquad (1)$$

The Symmetrization Postulate... ... ... 2/2

- The symmetrization postulate for a system of $n-$ identical particles sates that the total wave function must be symmetric under simultaneous exchange of variables ξ_j and ξ_k for every pair j, k, if the particles are bosons and the relation

$$\psi(\xi_1, \cdot\xi_j \cdots, \xi_k, \cdots, \xi_n) = +\psi(\xi_1, \cdot\xi_k, \cdots, \xi_j, \cdots, \xi_n) \quad (3)$$

should hold for all pairs jk. Similarly, for a system of $n-$ identical fermions the total wave function must be anti-symmetric under simultaneous exchange of variables ξ_j, ξ_k for every pair j, k

$$\psi(\xi_1, \cdot\xi_j \cdots, \xi_k, \cdots, \xi_n) = -\psi(\xi_1, \cdot\xi_k, \cdots, \xi_j, \cdots, \xi_n) \quad (4)$$

For a system of several identical bosons, the total wave function $\Psi(\xi_1, \cdots, \xi_n)$ remains unchanged under an arbitrary permutation of $\xi_1, \cdots, \xi_n$; where as for fermions the wave function remains unchanged under an even permutation but

Remarks 1/2

We now give some explanatory remarks on the symmetrization postulate.

1. The postulate is a statement about the **full** wave function of the system of identical particles under a **simultaneous exchange of all** the variables. For example, there is no constraint on the space part (or the spin part) of the wave function alone.
2. For composite systems such those consisting of both bosons and fermions, the symmetry requirement shold for every pair of identical bosons and identical fermions separately.
3. For a system consisting of several 'particles', which themselves could be bound state of bosons and fermions, the postulate applies with spin interpreted to mean the total angular momentum in the centre of mass frame.

Remarks 2/2

4. While for a system of two particles the symmetry property is restricted to symmetry or antisymmetry alone, for a system of many identical particles theoretical considerations allow existence of a variety of possibilities under permutation of variables. These choices, known generally as 'para-statistics', do not seem to play any role for real physical systems.

Spin Statistics Connection

Our discussion of the symmetrization postulate will be incomplete if we do not mention the spin statistics connection contained in the postulates is contained the symmetrization postulate has been proved by Pauli and Luders within the framework of relativistic quantum field theory under very general assumptions such as relativistic invariance, micro causality and positivity of the Hamiltonian.

A Symmetry Property of Particular Two Particle States

Here give two useful statements which will be needed repeatedly in connection implementation of the symmetrization postulate.

- ✍ When we add two, equal, angular momenta j the possible resulting values are $J = 2j, 2j-1, \cdots, 0$. Of these the state with the highest value, $J = 2j$, is symmetric under an exchange of the two particles, the next one, with $2j-1$ is antisymmetric; the states being alternately symmetric and antisymmetric as J takes on the values in descending order.
- ✍ For a two particle system, the effect of an exchange of the positions of the two particles is same as the parity on the wave function in the centre of mass frame. Therefore under an exchange of the space variables the space part of the wave function is symmetric for even ℓ and antisymmetric for odd ℓ, where ℓ is the relative angular momentum.

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