Notes for Lectures in Quantum Mechanics *

Identical Particles in Quantum Mechanics

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1 A Short Review of Classical Concepts

In transition form classical to quantum mechanics one is forced to revise, or completely give up, many classical concepts. Some examples are

- Superposition principle is new for particles.
- Precise values for observables is given up. in general only probabilities can be computed.
- Not all observables can be measured simultaneously. Only those represented by a simultaneous commuting set can be measured simultaneously.

2 Indishtinguishability in QM

We shall see that for a system of two or more identical particles, the identity of individual particles looses its meaning in quantum mechanics; one can only talk about the system as a whole. We also need to introduce a new hypothesis about the states of a quantum mechanical system of several identical particles.

Classically it is always possible to distinguish two electrons because they are point particles having well defined positions. Each particle has a well

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defined trajectories which can be measured, and can also can be predicted if the interactions are known. Thus a distinction can always be maintained at all times by measuring the position of each particle at successive times at regular intervals very accurately and doing this does not disturb their motion.

3 Identical Particles in QM

For a quantum mechanical system consisting of several identical particles we note the following.

- In the first place the particles cannot have well defined positions, only some probabilities can be assigned to different values of the position.
- At any given time, it is certainly possible to localize each particle with great accuracy. However, in general, the wave packets spread and it will not be possible to maintain the localization at later times. Thus when the particles come very close we would not be able to tell which particle was which one.
- The trajectories of particles do not have any meaning in quantum mechanics. One may attempt to follow the motion in quantum mechanics by measuring their positions very accurately at short intervals; but this exercise turns out to be useless for all practical purposes because it 'disturbs' the motion of the particles.
- It is important to realize that inability to distinguish between identical particles is not due to some technical, theoretical or experimental, reasons. Even in principle a distinction between two identical particles cannot be maintained by a thought experiment or any theoretical an analysis. Nor is this conclusion avoidable by improving upon the measuring apparatus. This surprising conclusion is more due to the structure of the quantum theory, rather than lack of a very accurate measuring apparatus.
- In some sense the nature of the conclusion is very similar to the uncertainty principle about impossibility of very precise simultaneous measurement of position and momentum. We , therefore, wish to conclude

that even the possibility of being always able to distinguish between two identical particles does not exist even in principle and that we should accept that they are **indistinguishable**.

4 Loss of Identity

In view of the above discussion, we abandon the attempts to distinguish between identical particles. We *assume* that in a quantum mechanical system of identical particles it is not possible to distinguish between any two identical particles; the individual particles loose their identity and we should refer to the system as a 'whole'.

This has far reaching consequences. for sake of definiteness, let us consider a system of two electrons and let their wave function be $\psi(\xi_1, \xi_2)$ where ξ_1, ξ_2 collectively denote the space as well as the spin variables of the two electrons. Then $\psi(\xi_2, \xi_1)$ will denote the state of two electrons obtained by exchanging the two electrons. *It will be assumed now on wards that the wave functions $\psi(\xi_1, \xi_2)$ and $\xi(xi_2, \xi_1)$ represent the same state.*

5 Symmetric and Antisymmetric Wave Functions

If the two electrons are indistinguishable, this interchange of two electrons can have no effect on the state of the system as whole. Thus the wave functions $\psi(\xi_1, \xi_2)$ and $\psi(\xi_2, \xi_1)$ must represent the same state and hence we must have

$$\psi(\xi_1, \xi_2) = \exp(i\alpha)\psi(\xi_2, \xi_1) \tag{1}$$

for some real α, as required by the first postulate.

The above equation is valid for all ξ_1, ξ_2 and hence, replacing ξ_1 with ξ_2 and ξ_2 with ξ_1 in (1) we get

$$\begin{aligned} \psi(\xi_2, \xi_1) &= \exp(i\alpha)\psi(\xi_1, \xi_2) && (2) \\ &= \exp(2i\alpha)\psi(\xi_2, \xi_3) && (3) \end{aligned}$$

and hence we conclude that

$$\exp(2i\alpha) = 1 \qquad \Rightarrow \qquad \exp(i\alpha) = \pm 1. \tag{4}$$

Therefore, we arrive at an important conclusion that under exchange of all variables the wave function of two identical particles must be symmetric or antisymmetric

$$\psi(\xi_2, \xi_1) = \pm\psi(\xi_1, \xi_2). \tag{5}$$

6 Symmetry Property is Preserved in Time

The symmetric or the antisymmetric nature Eq.(5), if enforced at initial time, will be preserved at all times. To see this let us introduce a permutation operator P_{12} by

$$P_{12}\psi(\xi_1, \xi_2) = \psi(\xi_2, \xi_1) \tag{6}$$

Eq.(5) is just the statement that the state must be an eigenstate of the permutation operator. The Hamiltonian for two identical particles will be symmetric under exchange of ξ_1 and ξ_2 and hence commutes with P_{12} implying that the permutation operator is a constant of motion. Thus the wave function will remain an eigenfunction of P_{12} with the same eigenvalue at all times if it is chosen to be eigenfunction at initial time.