Notes for Lectures in Quantum Mechanics [1]

Spin Wave Function

A. K. Kapoor
http://0space.org/users/kapoor
akkapoor@cmi.ac.in; akkhcu@gmail.com

October 27, 2021

[1] Updated:Oct 23, , 2021; Ver 0.x

Overview

0.1. Wave function of a particle with spin ... 1/2

0.2. Spin part of wave function

QM of Particle with Spin

We have so far discussed a quantum description of the spin degrees of freedom of a particle. The other dynamical variables of a particle are the usual coordinates, momenta, etc. It is assumed that the spin and position are independent of each other and hence can be measured simultaneously. Similarly spin and momentum can be measured simultaneously. Thus the spin operators $\vec{S}$ commute with position operators and also with the momentum operators. In such a space a representation could be chosen in which the basis vectors are simultaneous eigenvectors of $\vec{S}^2, S_z$, and $\vec{r}$ operators. Denoting a basis vector as $|\vec{r}\rangle|sm\rangle$, an arbitrary vector will have an expansion

$$|\psi\rangle = \sum_m \int dx \, C_{m\vec{r}}|\vec{r}\rangle|sm\rangle \tag{1}$$

Wave function of a particle with spin ... 1/2

The coefficients $C_{m\vec{r}}$ give the probability amplitude of position begin $\vec{r}$ and S_z having a value m, and just the $(2s+1)$ component functions. By a change in notation we write the $(2s+1)$ component wave function as

$$\Psi(\vec{r}) = \begin{pmatrix} \psi_1(\vec{r}) \\ \psi_1(\vec{r}) \\ \vdots \end{pmatrix} \tag{2}$$

In this representation the state of a particle with spin is described by a vector in the vector space which is tensor product of a complex vector space of dimension $(2s+1)$ and the space of square integrable functions. In particular a particle with spin $\frac{1}{2}$, such as an electron, is described by a two component wave function

$$\Psi(\vec{r}) = \begin{pmatrix} \psi_1(\vec{r}) \\ \psi_1(\vec{r}) \end{pmatrix} \tag{3}$$

Wave function of a particle with spin ... 2/2

The interpretation of the different components of Ψ is, that $|\psi_1(\vec{r})|^2 d^3 r$ gives the probability of spin being *up* and position being between $\vec{r}$ and $\vec{r} + d\vec{r}$. Similarly, $|\psi_2(\vec{r})|^2 d^3 r$ gives the probability of spin being *down* and position being between $\vec{r}$ and $\vec{r} + d\vec{r}$. The normalization condition now reads

$$\int \Psi^\dagger(\vec{r})\,\Psi(\vec{r}) d^3\vec{r} = \int \left(|\psi_1(\vec{r})|^2 + |\psi_2(\vec{r})|^2 \right) d^3\vec{r} = 1 \qquad (4)$$

Spin part fo wave function

Frequently, the total wave function factorizes and assumes the form

$$\Psi(\vec{r}) = \psi(\vec{r}) \times \chi \tag{5}$$

where χ is a column vector with $(2s+1)$ components, so for an electron we will have

$$\chi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}. \tag{6}$$

We shall then refer to $\psi(\vec{r})$ as the `space part` of the wave function and the column vector χ as the `spin part` of the wave function. The function $\psi(\vec{r})$ describes the translational degrees of freedom, as usual, and the spin degrees of freedom are described by the column vector χ.

qm-lec-20003 0.x Created : , 2021 Printed : October 27, 2021 KApoor

PROOFS

Open LaTeX File qm-lec-20003