Notes for Lectures in Quantum Mechanics *

Spin as a Dynamical Variable

(Your Participation is Important)

A. K. Kapoor
http://0space.org/users/kapoor
akkapoor@cmi.ac.in; akkhcu@gmail.com

Contents

1 What is spin?

- Spin of a particle is its angular momentum at rest.
- For a composite system, such as a nucleus, it is the value of angular momentum in the center of mass frame.
- For a classical point particle the angular momentum, given by $\vec{r}\times\vec{p}$, is zero when the particle is at rest. Therefore spin has no classical analogue.

2 What led to discovery of spin?

In order to explain anomalous Zeeman effect it was suggested by Goudsmidt and Uhlenbeck that electron possesses angular momentum at rest whose component in any fixed direction can take one of the two values $\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$). Associated with spin there is a magnetic moment, of one negative Bohr magneton, given by

$$\vec{\mu} = -\frac{e}{mc}\vec{S}$$

*Updated:JUl 12, 2021; Ver 0.x

Many elementary particles are found to have angular momentum at rest. This angular momentum is called *spin*.

3 How is spin described in quantum mechanics?

Spin is an observable associated with all the fundamental particles, having the same properties as the angular momentum. Therefore, we associate three operators S_x, S_y, and S_z with spin angular momentum and assume that they satisfy angular momentum algebra.

$$[S_x, S_y] = i\hbar S_z \quad (1)$$
$$[S_y, S_z] = i\hbar S_x \quad (2)$$
$$[S_z, S_x] = i\hbar S_y \quad (3)$$

4 Properties of spin operators

The commutation relations of spin operators imply that the operator $\vec{S}^2 = S_x^2 + S_y^2 + S_z^2$ commutes with all the three components of spin. Since different components of spin do not commute, a commuting set of operators has $\vec{S}^2$ and components of the spin along any one direction; most common choice being $\vec{S}^2$ and S_z.

The results on angular momentum apply to the spin also and we have

- The eigenvalues of $\vec{S}^2$ are given by $s(s+1)\hbar^2$ where s is a positive integer of half integer.
- For a given value of s, the eigenvalues of S_z are $s, s-1, s-2, \ldots, -s$.
- A particle will be said to have spin s if the the maximum allowed value of S_z is $s\hbar$, which is same as $\vec{S}^2$ having value $s(s+1)\hbar^2$.

A simultaneous eigenvector of $\vec{S}^2$ and S_z will be denoted by $|sm\rangle$ which will have the properties

$$\vec{S}^2|sm\rangle = s(s+1)\hbar^2|sm\rangle \quad (4)$$
$$\vec{S}_z|sm\rangle = m\hbar|sm\rangle \quad (5)$$

In all there are $(2s+1)$ values of m ranging from $-s$ to s and therefore $(2s+1)$ eigenvectors $|sm\rangle$. The vector space needed to describe spin is linear span of all the vectors $|sm\rangle$ and is $(2s+1)$ dimensional.