Notes for Lectures on Quantum Mechanics *

Green Function for Perturbative Solution of Scattering

A. K. Kapoor
http://0space.org/users/kapoor
akkapoor@cmi.ac.in; akkhcu@gmail.com

For the scattering problem we need to set up an integral equation for the Schrodinger equation.

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi(\vec{r}) + V(\vec{r})\psi(\vec{r}) = E\psi(\vec{r}). \tag{1}$$

We write the Schrodinger equation as

$$\big(\nabla^2\psi(\vec{r}) + k^2\psi(\vec{r})\big) = \frac{2\mu}{\hbar^2}V(\vec{r})\psi(\vec{r}), \tag{2}$$

where $k^2 = \sqrt{\frac{2\mu E}{\hbar^2}}$, and introduce the Green function as solution of the equation

$$\big(\nabla^2\psi(\vec{r}) + k^2\psi(\vec{r})\big)G(\vec{r}) = -\delta(\vec{r}). \tag{3}$$

The solutions for the Green function can be obtained using Fourier transform method and are given by 18002

$$G_\pm(\vec{r}) = \frac{e^{\pm ikr}}{4\pi r}. \tag{4}$$

Note that the Green function introduced here is for `free particle Schrodinger equation` (2). This is because we wish to set up an integral equation for perturbative solution of the scattering problem. It turns out that the Green function $G_+(\vec{r})$ 18003 gives solution obeying correct boundary condition for scattering problem.

qm-lecs-18011 0.x Created : Oct 26, 2021 Printed : October 26, 2021 KApoor

* qm-lecs-18011 Updated:Oct 26, 2021; Ver 0.x