Notes for Lectures in Quantum Mechanics *

Validity of Born Approximation

Square Well Potential

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For a square well potential of strength V_0 and range R_0, the expression in the left hand side Eq.(**??**) takes the form

$$\frac{\mu V_0}{\hbar^2 k}\left|\int_0^{R_0}\left(e^{2ikr}-1\right)dr\right| = \frac{\mu V_0}{\hbar^2 k}\left|\frac{e^{2ikR_0}-1}{2ik}-R_0\right| \tag{1}$$

$$= \frac{\mu V_0}{2\hbar^2 k^2}\left|e^{2ikR_0}-2ikR_0-1\right| \tag{2}$$

Using the notation $\rho = 2kR_0$ the condition, that the Born approximation be valid, takes the form

$$\frac{\mu V_0}{2\hbar^2 k^2}\left(\rho^2 - 2\rho\sin\rho - 2\cos\rho + 2\right)^{\frac{1}{2}} << \tag{3}$$

we shall consider the low energy and high energy cases separately.

Low energy scattering

At low energy the de Broglie wave length is much larger than the range of the potential *i.e.* $2kR_0 << 1$. We then have

$$\begin{aligned}\rho^2 &- 2\rho\sin\rho + 2\cos\rho \\ &\approx \rho^2 - 2\rho\left(\rho - \frac{\rho^3}{6}+\cdots)\right) - 2\left(1-\frac{\rho^2}{2}+\frac{\rho^4}{24}+\cdots\right)+2 \\ &= \frac{\rho^4}{4}\end{aligned} \tag{4}$$

Hence at low energies the Born approximation is applicable if

$$\frac{\mu V_0}{2\hbar^2 k^2}\frac{\rho^2}{2} = \frac{\mu V_0 R_0^2}{\hbar^2} << 1 \tag{5}$$

The above condition implies that the potential is so weak that the bound state does not exist.

High energy limit

In the high energy limit $\rho >> 1$ and we get

$$\left(\rho^2 - 2\rho\sin\rho - 2\cos\rho + 2\right)^{\frac{1}{2}} \approx \rho \tag{6}$$

*Updated:Jul 9 2021; Ver 0.x

and hence the Born approximation is valid if

$$\frac{\mu V_0}{2\hbar^2 k^2}\rho << 1 \tag{7}$$

or

$$\frac{\mu V_0 a}{\hbar v} << 1 \tag{8}$$