# Notes for Lectures in Quantum Mechanics *

## Green Function for Poisson Equation

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In electromagnetic theory the electric potential satisfies the Poisson equation

$$\nabla^2\Phi = -\frac{\rho}{\varepsilon_0}, \tag{1}$$

where $\rho(\vec{r})$ is the volume charge density. The Green function for the Poisson equation is defined by

$$\nabla^2 G(\vec{r}) = -\delta^3(\vec{r}) \tag{2}$$

If $\phi_0(\vec{r})$ is a solution of the Laplace equation

$$\nabla^2\phi_0(\vec{r}) = 0, \tag{3}$$

then

$$\Phi(\vec{r}) = \phi_0(\vec{r}) + \int G(\vec{r}-\vec{r}')\frac{\rho(\vec{r}')}{\varepsilon_0}d^3r' \tag{4}$$

is a solution of the Poisson equation which can be easily verified by applying $\nabla^2$ on both sides of Eq.(4).

$$\begin{aligned}
\nabla^2\Phi(\vec{r}) &= \nabla^2\phi_0(\vec{r}) + \nabla^2\int G(\vec{r}-\vec{r}')\frac{\rho(\vec{r}')}{\varepsilon_0}d^3r' &(5)\\
&= \int \nabla^2 G(\vec{r}-\vec{r}')\frac{\rho(\vec{r}')}{\varepsilon_0}d^3r' &(6)\\
&= -\frac{1}{\varepsilon_0}\int \delta^3(\vec{r}-\vec{r}')\rho(\vec{r}')d^3\vec{r}' &(7)\\
&= -\frac{\rho(\vec{r})}{\varepsilon_0} &(8)
\end{aligned}$$

It can be shown that one solution of Eq.(2) is

$$G(\vec{r}) = \frac{1}{4\pi r}. \tag{9}$$

This Green function gives the potential due to a charge distribution subject to the condition that the potential vanishes at infinity. The exact form of the Green function and the solution $\phi_0$ of the Laplace equation is determined by the boundary conditions of the problem.

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