Notes for Lectures on Quantum Mechanics

Some Useful Restrictions on CG coefficients

t

A. K. Kapoor
http://0space.org/users/kapoor
akkapoor@cmi.ac.in; akkhcu@gmail.com

In this connection with addition of angular momenta, the following results from the theory of angular momentum derived earlier will be useful.

$$J_\pm|JM\rangle = \sqrt{(J(J+1) - M(M\pm 1)}|JM\pm 1\rangle \tag{1}$$

$$\begin{aligned}(J_\pm^{(1)} + J_\pm^{(2)})|j_1m_1j_2m_2\rangle &= \sqrt{(j_1(j_1+1) - (m_1\pm 1)}|j_1m_1\pm 1j_2m_2\rangle \\ &+ \sqrt{(j_2(j_2+1) - m_2(m_2\pm 1))}|j_1m_1j_2m_2\pm 1\rangle\end{aligned} \tag{2}$$

On taking conjugate of Eq.(2) we get

$$\begin{aligned}\langle j_1m_1j_2m_2|(J_\mp^{(1)} + J_\mp^{(2)}) &= \langle j_1(m_1\pm 1)j_2m_2|\sqrt{(j_1(j_1+1) - (m_1\pm 1)} \\ &+\langle j_1m_1j_2(m_2\pm 1)|\sqrt{j_2(j_2+1) - m_2(m_2\pm 1))}\end{aligned} \tag{3}$$

which is a consequence of the angular momentum commutation relations. Considering the matrix element

$$\langle j_1j_2m_1m_2|J_\pm|JM\rangle = \langle j_1j_2m_1m_2|(J_\pm^{(1)} + J_\pm^{(2)})|JM\rangle \tag{4}$$

and using Eq.(1) and Eq.(3) we get two relations, one for J_+ and

$$\begin{aligned}&\sqrt{J(J+1) - M(M+1)}\langle j_1j_2m_1m_2|J(M+1)\rangle \\ &= \langle j_1j_2m_1 - 1m_2|JM\rangle\sqrt{j_1(j_1+1) - m_1(m_1+1)} \\ &\quad +\langle j_1j_2m_1m_2 - 1|JM\rangle\sqrt{j_2(j_2+1) - m_2(m_2+1)}\end{aligned} \tag{5}$$

and a second relation for J_-

$$\begin{aligned}&\sqrt{J(J+1) - M(M-1)}\langle j_1m_1, j_2m_2|J(M-1)\rangle \\ &= \langle j_1(m_1+1)j_2m_2|JM\rangle\sqrt{j_1(j_1+1) - m_1(m_1-1)} \\ &\quad +\langle j_1m_1, j_2(m_2+1)|JM\rangle\sqrt{j_2(j_2+1) - m_2(m_2-1)}\end{aligned} \tag{6}$$

We will make repeated use of the results Eq.(5),Eq.(6) given above. These equations can be used successively with $M = J, J-1, \ldots$ to compute the Clebsch Gordon coefficients.

qm-lec-17003.pdf Ver 16.x
Printed : October 23, 2021
Created : Jul 2016
KAPOOR