Lecture on Quantum Mechanics *

Particle in A Box and in Square Well

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The energy levels of a particle in one dimensional infinite well

$$V(x) = \begin{cases} 0, & 0 \le x \le L \\ \infty & \text{outside} \end{cases} \tag{1}$$

can be found by solving the Schrödinger equation for $0 \le x \le L$.

$$-\frac{\hbar^2}{2m}\frac{d^2u(x)}{dx^2} = Eu(x), \qquad 0 \le x \le L. \tag{2}$$

Here the particle is like a free particle and the most general solution can written as

$$u(x) = A\sin kx + B\cos kx, \qquad k^2 = \frac{2mE}{\hbar^2}. \tag{3}$$

Out side the box, the potential is infinity and the solution vanishes:

$$u(x) = 0, \qquad \text{if } x < 0, \text{ or } x > L. \tag{4}$$

The boundary conditions to be imposed on the solution are

$$u(0) = u(L) = 0, \tag{5}$$

and no restriction on the derivatives at the boundary points $x = 0, x = L$. This gives

$$u(0) = 0 \quad \Rightarrow \quad B = 0, \tag{6}$$
$$u(L) = 0 \quad \Rightarrow \quad \sin kL = 0. \tag{7}$$

because $A \ne 0$. The solutions of this equation are $k_n = n\pi/L, n = 1, 2, \ldots$. The energy levels are given by

$$E_n = \frac{\hbar^2 k_n^2}{2m}, = \frac{\hbar^2 n^2 \pi^2}{2mL^2} \tag{8}$$

and the corresponding normalized wave functions are

$$u_n(x) = \begin{cases} \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) & 0 \le x \le L \\ 0 & x < 0 \text{ or } x > L. \end{cases} \tag{9}$$

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and n takes all positive integral values. *It should be noted that for $k = 0$ the solution vanishes identically and therefore $n = 0$ is unacceptable.* The wave functions in (9) obey the normalization

$$\int_0^L u_n(x) u_m(x)\, dx = \delta_{mn}. \tag{10}$$