Notes for Lectures on Quantum Mechanics *

A Summary of Coordinate and Momentum Representation

A. K. Kapoor
http://0space.org/users/kapoor
akkapoor@cmi.ac.in; akkhcu@gmail.com

In many problems it is useful to work with **the momentum representation**. In the momentum representation the orthonormal basis is chosen to be the eigenvectors of momentum operator(s) and roles of position and momentum get interchanged. The following table summarizes the coordinate and momentum representations. bigskipamount3mm

Comparison of coordinate and momentum representations

Item	**Coordinate representation**	**Momentum representation**
Choice of basis Vectors	$\vert\vec{r}\rangle$ are simultaneous eigenvectors of position operators.	$\vert\vec{p}\rangle$ are simultaneous eigenvectors of momentum operators.
Orthogonality	$\langle\vec{r}''\vert\vec{r}'\rangle = \delta(\vec{r}' - \vec{r})$	$\langle\vec{p}''\vert\vec{p}'\rangle = \delta(\vec{p}' - \vec{p})$
Completeness formula	$\int d^3r\vert\vec{r}\rangle\langle\vec{r}\vert = \hat{I}$	$\int d^3p\vert\vec{p}\rangle\langle\vec{p}\vert = \hat{I}$
Expansion in the basis	$\vert\psi\rangle = \int d^3r\vert\vec{r}\rangle\langle\vec{r}\vert\psi\rangle$	$\vert\psi\rangle = \int d^3r\vert\vec{p}\rangle\langle\vec{p}\vert\psi\rangle$
State representative	Coordinate space wave function $\tilde{\psi}(\vec{r}) \equiv \langle\vec{r}\vert\psi\rangle$	Momentum space wave function $\tilde{\psi}(\vec{p}) \equiv \langle\vec{p}\vert\psi\rangle$
Norm squared :$\Vert\psi\Vert^2$	$\langle\psi\vert\psi\rangle = \int \vert\psi(\vec{r})\vert^2 d^3r$	$\langle\psi\vert\psi\rangle = \int \vert\tilde{\psi}(\vec{p})\vert^2 d^3p$
Scalar product $\langle\psi\vert\phi\rangle$	$\int \psi(\vec{r})^*\phi(\vec{r})d^3r$	$\int \psi(\vec{p})^*\phi(\vec{p})d^3r$
Action of position operator $\hat{x}$	$\hat{x}\psi(\vec{r}) = x\psi(\vec{r})$	$\hat{x}\tilde{\psi}(p) = i\hbar\dfrac{\partial}{\partial p_x}\tilde{\psi}(p)$
Action of momentum operator $\hat{p}_x$	$\hat{p}_x\psi(\vec{r}) = -i\hbar\dfrac{\partial}{\partial x}\psi(\vec{r})$	$\hat{p}\tilde{\psi}(\vec{p}) = p_x\tilde{\psi}(\vec{p})$
Operator for $F(q_j, p_k)$	$\widehat{F}\Big(\hat{q}_j, -i\hbar\dfrac{\partial}{\partial q_k}\Big)$	$\widehat{F}\Big(i\hbar\dfrac{\partial}{\partial p_j}, p_k\Big)$

It must be noted that two different can be written for the same classical dynamical variable. Classically, pq and qp are the same functions but the operators $\hat{p}\hat{q}$ and $\hat{q}\hat{p}$ are different. So which operator should correspond to the classical product pq?

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Should it be $\hat{p}\hat{q}$ or $\hat{q}\hat{p}$? This is known as *ordering problem.* The only guideline available is that the operator corresponding to a dynamical variable must be hermitian. So, in case of pq, we must select

$$pq \rightarrow \frac{1}{2}(\hat{p}\hat{q} + \hat{p}\hat{q})$$

For more complicated expressions the requirement of hermiticity is not sufficient to get a unique answer.