Notes for Lectures on Quantum Mechanics *

A Summary of Time Evolution in Schrodinger Picture

A. K. Kapoor
http://0space.org/users/kapoor
akkapoor@cmi.ac.in; akkhcu@gmail.com

- Given the state of the system at a time t_0, the state vector at any other time is related to it by a unitary transformation $U(t, t_0)$.

$$|\psi t\rangle = U(t, t_0)\,|\psi t_0\rangle$$

- The equation of motion of quantum system is the Schrodinger equation

$$i\hbar \frac{d}{dt}\,|\psi t\rangle = \hat{H}|\psi t\rangle$$

where $\hat{H}$ is the Hamltonian operator of the system.

- The time evolution operator satisfies the equation

$$i\hbar \frac{\partial}{\partial t} U(t, t_0)|\psi t_0\rangle = \hat{H}(t) U(t, t_0)$$

- If the Hamiltonian does not depend on time, the evolution operator is

$$U(t, t_0) = \exp[-i\hat{H}(t - t_0)/\hbar]$$

- The average value of a dynamical variable,$\hat{F}$, satisfies

$$\frac{d}{dt}\,\langle \hat{F}\rangle = \langle \frac{\partial \hat{F}}{\partial t}\rangle + \frac{1}{i\hbar}\langle\,[\hat{F}, \hat{H}]\,\rangle$$

- A dynamical variable is a constant of motion if it commutes with the Hamiltonian.
- The energy eigenstates of a system are staionary; they do not change with time. The state vector of a stationary state at any time is equal to the initial state vector multiplied by a numerical phase factor.
- The average value of a *constant of motion* G is independent of time in every possible state of the system including *nonstationary states.*
- The avearge value of *every* dynamical variable is independent of time in *stationary* states.

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