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Causality in QFT

Causality means cause precedes the effect. So if an event A is cause of

another event B, A must happen before B. In a relativistic setting this is often stated by saying that no signal cam travel faster than light.

If two events x, y have space like separation, then it is possible to find two different Lorentz frames such that in one frame A happens before B and in another frame B happens before A. Relativistic invariance demands that there cannot be no causal relationship between such events. In other words A cannot be cause of B and also B cannot be cause of A.

Let us now formulate causality in quantum field theory taking simplest example of a real scalar field. It can be decomposed into positive and negative frequency parts $\phi(x) = \phi^{(+)}(x) + \phi^{(-)}(x)$. The amplitude for creaton of particle at space time point x and being destroyed at y is given by $\langle 0|(\phi^{(-)}(y)\phi^{(+)}(x))|0\rangle$. This amplitude will be zero if $x^0 < y^0$, as the particle can get destroyed before it is created. So we get

$$\langle 0|(\phi^{(-)}(y)\phi^{(+)}(x))|0\rangle = 0 \qquad \text{if} x^0 < y^0. \tag{1}$$

If x, y are space like separated then there exists a frame in which $y^0 < x^0$ and in that frame the same process will be described by $\langle 0|(\phi^{(-)}(x)\phi^{(+)}(y))|0\rangle$. Relativistic invariance demands that these two amplitudes must be equal. So we get

$$\langle 0|(\phi^{(-)}(y)\phi^{(+)}(x))|0\rangle = \langle 0|(\phi^{(-)}(x)\phi^{(+)}(y))|0\rangle \tag{2}$$

or we have

$$\langle 0|\phi^{(-)}(y)\phi^{(+)}(x) - (\phi^{(-)}(x)\phi^{(+)}(y))|0\rangle = 0 \tag{3}$$

The above equation can be rewritten as

$$\langle 0|\phi(y)\phi(x) - \phi(x)\phi(y))|0\rangle = 0. \tag{4}$$

Since the commutator of $[\phi(x), \phi(y)]$ is a c- number $\Delta(x-y)$, the above expression is simply equal to $i\Delta(x-y)$ Thus we get the result that the commutator $\big[\phi(x), \phi(y)\big]$ must vanish at large distances.

Another way of understanding the vanishing of commutators is as follows. A hermitian scalar field is an observable. In general a measurement disturbs measurement of other observables if they do not commute. Such a disturbance due to a measurement cannot travel faster than light. Thus a simultaneous measurement of $\phi(x), \phi(y)$ should be possible for space like separations. Hence the real scalar fields should commute for space like separations.

For fermions the field are not hermitian and there no reason to demand that they should commute at space like separations. However tone can construct hermitian bilinears of fermions and we should expect them to commute at space like separations.

All this is consistent with quantization of integral spin fields with commutators and for fermions with anticommutators. This result is the conclusion of spin statistics connection proved by first by Pauli under certain general assumptions. Later it has been proved by several authors with varying degree of rigour.

What has been given here is to provide an intuitive understanding in the spin statics theorem. Our discussion here is inspired by Bogoliubov and Shirkov. For more details see

N.N. Bogoliubov and D. V. Shrikov, §10.2, §10.3.
Introduction to Theory of Quantized Fields, 3rd Ed, John Wile and Sons Inc (1980).