QFT-17 Problem Sessions

Unit-02 Classical Fields

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1 Exercise::Action Principle

In this set x collectively denotes all components of a four vector $x = (x^0, x^1, x^2, x^3)$.

[1] The Lagranian density for a real scalar field is given to be

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi(x) \partial^\mu \phi(x) - \frac{m^2}{2}\phi^2(x) - \frac{g}{4}\phi^4(x).$$

[2] A massive spin one boson is like photon with mass. It is described by a four vector field V_μ. For a set of three massive vector bosons, collectively denoted as $\vec{V}_\mu = \{V_{1\mu}, V_{2\mu}, V_{3\mu}\}$, the Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4}\vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu} + \frac{1}{2}M^2 \vec{V}_\mu \cdot \vec{V}_\mu.$$

where

$$\vec{F}_{\mu\nu} = \partial_\mu \vec{V}_\nu - \partial_\nu \vec{V}_\mu + g\vec{V}_\mu \times \vec{V}_\nu.$$

Here operations dot $\cdot$, and cross $\times$, denote usual scalar and cross products of three vectors. Obtain the equation of motion. Derive equation of motion for $V_{j\alpha}$ and show that the equations of motion can be written as

$$\partial_\nu \vec{F}^{\nu\alpha} + g\big(\vec{F}^{\nu\alpha} \times \vec{V}_\alpha\big) + M^2 V^\alpha = 0.$$

[3] Prove that the following two Lagrangian densities $\mathscr{L}, \mathscr{L}'$ give the same equations of motion.

$$\mathscr{L} = \mathscr{L}(\partial_\mu \phi, \phi), \quad \text{and} \mathscr{L}' = \mathscr{L} + \partial_\mu \Omega, \tag{1}$$

where $\Omega \equiv \Omega(\phi)$ depends on $\phi(x)$only

2 Exercise::Hamiltonian Dynamics

[1] The Lagrangian density for nonlinear Schrodinger field is

$$\mathscr{L} = i\hbar\psi^*(x,t)\partial_t\psi(x,t) - \frac{\hbar^2}{2m}\partial_x\psi^*(x,t)\partial_x\psi(x,t) - \psi^*(x,t)V(x).\psi(x,t)$$

(a) Find the cannonical momentum conjugate to $\psi(x)$ and the Hamiltonian for the Schrodinger field.

(b) Obtain the Hamilton's equations of motion and verify that they imply the time dependent Schrodinger equation.

(c) Use Poisson brackets to show that

$$N = \int d^3x\psi^*(x,t)\psi(x,t)$$

is a constant of motion.

[2] The Lagrangian density for a complex Klein Gordon field $\phi(x)$ is given to be

$$\mathscr{L} = \partial_\mu\partial^\mu\phi^*(x)\phi(x) + m^2\phi(x) * 2\phi(x) + \lambda(\phi^*(x)\phi(x))^2$$

Find expression for the Hamiltonian, obtain the Hamiltonian equations of motion. Here x collectively denotes the four space time components of x^μ.