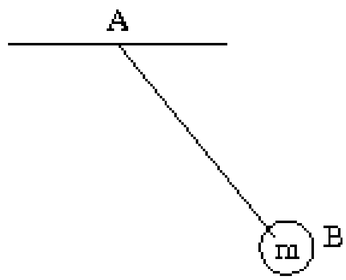
CHENNAI MATHEMATICAL INSTITUTE

CLASSICAL MECHANICS I

PROBLEM SHEET XII

14th November 2012
date due 21st November 2012



56. A mass m is attached to a massless spring (spring constant K , unstretched length L) and is free to oscillate in a vertical plane about A. Write the Lagrangian and the Hamiltonian of the system. Write both the Euler-Lagrange equation and the Hamilton's equation and show that they are identical.

57. Let the infinitesimal distance in a curved space be given by

$$ds^2 \;=\; (g_{ab}(q_1, q_2, \ldots, q_N))d\dot{q}_a d\dot{q}_b\, dt^2 \;=\; T^2 dt^2$$

where the summation over a and b is implied and take values 1,2,. . .,N and t is a parameter specifying the trajectory. Find the geodesic equation by minimising $F(T) = T^2$ - this is the same as minimising T as T^2 is a monotonic function of T. This saves considerable algebra. Show that the geodesic equation is

$$\ddot{q}_p \;=\; \frac{1}{2} g^{-1}_{pa} [\frac{\partial g_{cd}}{\partial q_a} - \frac{\partial g_{ad}}{\partial q_c} - \frac{\partial g_{ac}}{\partial q_d}] d\dot{q}_c d\dot{q}_d$$

(again the summation convention has been used)

58. A particle of mass m moves on a spiral $x_3 = k\,\theta, \rho = R$ (we are using cylindrical co-ordinates (ρ, θ, x_3) , x_3 being the vertical direction).Find the Hamiltonian and solve the equations of motion.

59. Consider the Lagrangian of a charged particle (mass m , charge q) in the presence of an electric $\vec{E}(\vec{r},t)$ and a magnetic field $\vec{B}(\vec{r},t)$. The Lagrangian for the particle is given by

$$L \;=\; \frac{m\,\dot{\vec{r}}.\dot{\vec{r}}}{2} \;+\; q(-\phi(\vec{r},t) \;+\vec{A}(\dot{\vec{r}},t))$$

where

$$\vec{E}(\vec{r},t) \;=\; -\vec{\nabla}\,\phi(\vec{r},t) - \frac{\partial\vec{A}(\vec{r},t)}{\partial t}$$

and

$$\vec{B}(\vec{r},t) \;=\; \vec{\nabla}\,\times\,\vec{A}(\vec{r},t)$$

Show that this leads to the correct force law

$$m\frac{d^2\vec{r}}{dt^2} \;=\; q(\vec{E}(\vec{r},t) \;+\; \dot{\vec{r}}\,\times\,\vec{B}(\vec{r},t))$$

where $\vec{v} \;=\; \frac{d\vec{r}}{dt}$ is the velocity of the particle.

60. Consider a particle of mass m moving in a plane under the central force

$$F(r)\hat{r} \;=\; (-\frac{k}{r^2} \;+\; \frac{k'}{r^3})\hat{r}$$

with $k \;>\; 0$.

(a) Write the Lagrangian in terms of polar coordinates (r,θ).

(b) Find the constants of motion.

(c) Assume the angular momentum l^2 is greater than $-m\,k'$. Obtain an equation for $u \;=\; 1/r$ as a function of θ. Get the general solution for r as a function of θ.