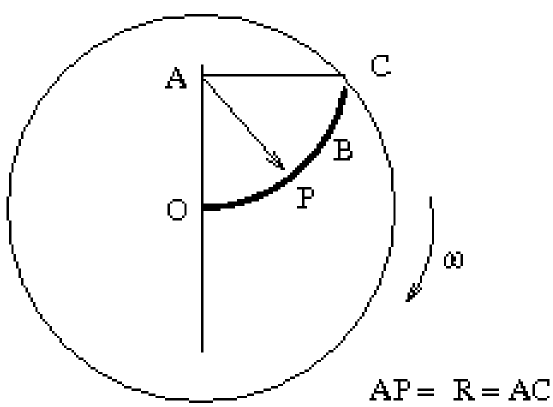
CHENNAI MATHEMATICAL INSTITUTE
CLASSICAL MECHANICS I
PROBLEM SHEET VIII

10th October 2012
date due 22nd October 2012

36. A projectile is fired vertically up with an initial velocity $\vec{U_0} = U_0\,\hat{\epsilon}_3$ from a town located at latitude λ. Find the place it will come back to the ground taking the earths rotation into account. (East is along $\hat{\epsilon}_1$ and north along $\hat{\epsilon}_2$.

37. A merry-go-round starts from rest and constantly accelerates at α revolutions per second per second. a person is siting at a distance of R metres from the axis of rotation and is holding a mass M kilograms. Calculate the force (vector!) the person must exert on the mass after time t. The person's co-ordinates at $t = 0$ is $R\hat{\epsilon_1}$ (in the ground frame) and the motion is counterclock-wise as seen from above.

38. A small body is placed on an incline which is pointed along the southern - northern direction. Find the angle of inclination below which the body will slide up the plane neglecting friction. Assume the incline is located at latitude λ.



39. A particle of mass m is moving frictionlessly along a circular slot OPBC of radius R. The slot is made on a disk (centre O) which is moving with a uniform angular velocity ω It starts from rest when the $\angle OAP = \theta = 0$ (that is from O).

(a) Find the force exerted by the slot on the particle as a function of θ.

(b) Find the speed u of the particle relative to the slot just as it reaches C

40. Consider a particle having an initial velocity $\vec{V_0}$ (say a bullet being fired from the earth's surface) at a latitude λ. Assuming the rotation of the is constant ($\frac{d\vec{\Omega}}{dt} = 0$) we have the equation for the motion of the particle is given by (in the rotating frame of the earth)

$$\frac{d^2\vec{r}}{dt^2} = -2(\vec{\Omega} \times \frac{d\vec{r}}{dt}) + \vec{g}_{eff} \quad \text{Equation(A)}$$

where

$$\vec{g}_{eff} = \vec{g} - \vec{\Omega} \times (\vec{\Omega} \times \vec{r})$$

(a) Show that in the rotating frame (show $-\vec{\nabla}V_{eff} = m\vec{g}_{eff}$), where V_{eff} is given by

$$V_{eff} = V - \frac{m}{2}(\vec{\Omega} \times \vec{r}).(\vec{\Omega} \times \vec{r})$$

and $V = -m\vec{g}.\vec{r}$.

(b) Show that

$$\vec{r}(t) + \vec{r}(t=0) + \vec{v}(t=0)t + \frac{\vec{g}_{eff} + 2\,(\vec{v}(t=0) \times \vec{\Omega})t^2}{2} + \frac{1}{3}(\vec{g}_{eff} \times \vec{\Omega})t^3$$

upto order $g\,\vec{\Omega}\,t^3$.

NOTE: If we assume $\vec{g}_{eff}$ is constant, Equation (A) can be solved exactly. The answer is

$$\vec{r}(t) = \vec{r}(t=0) + \vec{v}(t=0)t + \frac{\vec{g}_{eff}t^2}{2} + \frac{(\cos(2\Omega \mathrm{t}) - 1)}{2\Omega^2}(\vec{\Omega} \times \vec{v}_0)$$

$$-\frac{1 - 2\Omega^2\,t^2 - \cos(2\Omega \mathrm{t})}{4\Omega^4}[\vec{\Omega} \times (\vec{\Omega} \times \vec{g}_{eff})]$$

$$-\frac{2\Omega\,t - \sin(2\Omega \mathrm{t})}{2\,\Omega^3}[\frac{1}{2}(\vec{\Omega} \times \vec{g}_{eff}) - \vec{\Omega} \times (\vec{\Omega} \times \vec{v}(t=0)]$$

Try and get it (not for submission) Will be happy to discuss with you on how to obtain this expression.