CHENNAI MATHEMATICAL INSTITUTE

CLASSICAL MECHANICS I

PROBLEM SHEET III

27th August 2012
date due 3rd September
August 2012

11. An electron moving with an intital velocity enters an electric field $E(t)\hat{\epsilon}_1$ with a velocity $V_0 \cap \epsilon_1$. The acceleration is given by $\vec{a} = eE(t)\hat{\epsilon}_1/m$, where m is the mass of the electron. The magnitude of the electric field is given by

$$E(t) = 0 \; for \; t < 0; E(t) = \frac{E_0 t}{\tau} \; for 0 < t < \tau;$$
$$E(t) = \frac{E_0}{\tau}(t - \tau) \; for \, \tau < t < 2\tau;$$
$$E(t + 2\tau) = E(t) \; t > 0$$

Find the distance travelled in time $n\tau$, n being an integer..

12. Consider a damped harmonic oscillator described by

$$m\frac{d^2x}{dt^2} + \lambda\frac{dx}{dt} + k\,x = 0$$

with $x(t = 0) = A$ and $\frac{dx}{dt}|_{t=0} = 0$. Calclate the average value of the total energy $< E >_{nT}$ over a cycle $nT \leq t \leq (n+1)T$, after n cycles. T is the time period. Use the result to show

$$< E >_{nT} = < E >_0 \; e^{-2\gamma nT} \; for \; \gamma << (\frac{k}{m})^{1/2}$$

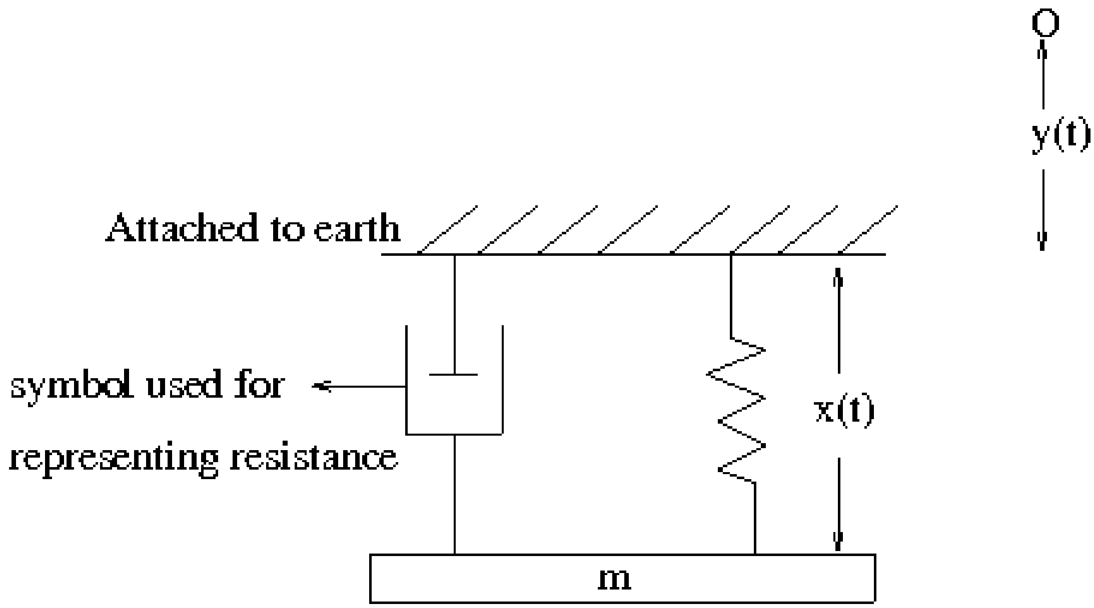
where $\gamma = \lambda/(2m)$.

13. Find the equilibrium position and the frequency of small oscillations about the equilibrium position for the potential

$$V(x) = -\frac{a}{x^6} + \frac{b}{x^{12}}$$

where a, b are positive constants.

14. The Seismograph: This instrument is used in recording disturbances within the earth, such as those created in earthquakes. The instrument is a spring mass system with a resistance term proportional to the velocity. See figure



Let $y(t)$ be the displacement of the platform with respect O, origin in an inertial frame. $x(t)$ is the displacement of the mass m with respect to the platform. If $y(t) = A\cos(\Omega \mathrm{t})$ show that

$$x(t) = \frac{mg}{k} + L + \frac{A\cos(\Omega \mathrm{t} + \phi)\Omega^2}{(\omega^2[(1 - \frac{\Omega^2}{\omega^2})^2 + \frac{4\gamma^2\Omega^2}{\omega^4}])^{1/2}}$$

where

$$\tan(\phi) = \frac{2\gamma\Omega}{\Omega^2 - \omega^2}$$

L, k is the natural length of the spring (massless) and the spring constant respectively , $\omega^2 = k/m$. If the spring is soft

$$\frac{\omega}{\Omega} \rightarrow 0$$

, show that when $\gamma/\Omega << 1$

$$x(t) \; = \; \frac{mg}{k} \; + \; L\,A\mathrm{cos}(\Omega \mathrm{t} + \phi)$$

Thus

$$\eta \; = \; x(t) \; - L \; - \frac{mg}{k} \; \propto \; A$$

the amplitude of the forced oscillation which is what the Seismograph measures..

Further if $\frac{\omega}{\Omega} \; >> \; 1$, $\eta \; \propto \; A\Omega^2$ which measures the acceleration fo the platform. Thsi is the principle of the accelerometers used in aircrafts.

15. Consider a mass m moving under a simple harmonic force (force constant $k \; = m\omega^2$) with dry friction (Force of friction = Normal reaction X coeffficient of friction). Taking the normal reaction as mg and μ as the coefficient of friction discuss the motion. Assume an initial state at $t = 0$ as $x(0) = A\,;\; \frac{dx}{dt}(0) = 0$. Consider A in the range $0.5\mu\, mg/k$ *to* $2.5\mu\, mg/k$.