SCHOOL OF BASIC SCIENCES,
INDIAN INSTITUTE OF TECHNOLOGY BHUBANESWAR

JOINT-M.Sc.-Ph.D.-PHYSICS (2016-2018)

PH5L008 : STATISTICAL MECHANICS

Semester : January - April, 2017

TEST - 1

08:00 - 10:00 hrs. (2 hours)

1 February 2017 (Wednesday)

- Begin answering a new question from a fresh page.
- Calculators are permitted
- Maximum Marks : 50
- Wishing you the very best ..... KPN & AKK

(1) There are N ideal gas molecules in a room of volume $V = 10$ M^3 and at room temperature. The molecules are in equilibrium. There is an imaginary box of volume v M^3 completely contained within the room. The walls of the box are permeable and conducting. In other words, the box exchanges energy and matter with its surroundings.

Let ν denote a random variable defined as the number of molecules in the box. Let $P(n)$ be the probability that $\nu = n$. Let $\mu = Nv/V$.

Derive appropriate expressions for $P(n)$ to describe the statistics of n for the following cases : (i) $v = 6$ M^3 and $N = 10$ (ii) $v = 10$ cm^3 and $N = 10^5$ Calculate the mean and variance of the random variable ν for both cases. **(12 Marks)**

(2) The canonical partition function function of an ideal gas of N mono-atomic point particles occupying a volume V and at temperature T is given by

$$Q(T,V,N) = \frac{V^N}{N!}\frac{1}{\Lambda^{3N}}, \quad \text{where } \Lambda = \frac{h}{\sqrt{2\pi m k_B T}}$$

Derive an expression for entropy as a function of T, V, and N. Consider infinitesimal change of S brought out by infinitesimal changes in V and T. All these changes are quasi static and reversible, The process is adiabatic if the change in entropy is zero. Solve the resulting differential equation and show that for a quasi static reversible adiabatic process, $TV^{2/3} = \Theta$, where Θ is a constant. **(20 Marks)**

(3) Show that in the canonical ensemble formalism, the entropy S of the system is related to the partition function Q as given below.

$$S = k_B \left[\ln Q + T \left(\frac{\partial \ln Q}{\partial T} \right)_V \right].$$

(6 Marks)

(4) In each of the following, pick up the right option and give a brief explanation/derivation.

[4.1] N molecules of an ideal gas are in a volume V with energy E. Keeping the energy constant, the volume is doubled. The entropy changes by,

(A) $N \ln 2$ (B) $N \ln(1/2)$ (C) $N^2 \ln 2$ (D) $-N^2 \ln(2)$ **(2 Marks)**

[4.2] Enthalpy of a simple fluid is a function of

(A) T, P, N (B) S, P, N (C) S, V, N (D) T, P, μ **(2 Marks)**

[4.3] Let T be the temperature and $\widehat{\Omega}$ the number of micro states of an isolated system. T and $\widehat{\Omega}$ are related by,

(A) $k_B T = \left(\frac{\partial \widehat{\Omega}}{\partial E} \right)_{V,N}$ (B) $k_B T = \left(\frac{\partial \ln \widehat{\Omega}}{\partial E} \right)_{V,N}$

(C) $\frac{1}{k_B T} = \left(\frac{\partial \widehat{\Omega}}{\partial E} \right)_{V,N}$ (D) $\frac{1}{k_B T} = \left(\frac{\partial \ln \widehat{\Omega}}{\partial E} \right)_{V,N}$ **(2 Marks)**

[4.4] The density of (energy) states $g(\epsilon)$, of a single-particle in a three dimension, modelled by micro canonical ensemble, depends on energy ϵ as,

(A) $\epsilon^{3/2}$ (B) $\epsilon^{1/2}$ (C) $\epsilon^{-1/2}$ (D) ϵ **(2 Marks)**

[4.5] An ideal gas expands adiabatically and reversibly from (V_1, P_1) to (V_2, P_2). The process is described by the equation $PV^\gamma = \Theta$, where γ and Θ are constants. During the expansion the system transacts certain amount of energy (W), by work with the surroundings. W is given by,

(A) $\frac{P_1 + P_2}{2}(V_1 - V_2)$ (B) $\frac{P_1 V_1 - P_2 V_2}{1 - \gamma}$

(C) $\frac{P_1(V_2^{1+\gamma} - V_1^{1+\gamma}}{1 + \gamma}$ (D) $\frac{P_2(V_2^{1+\gamma} - V_1^{1+\gamma}}{1 + \gamma}$ **(2 Marks)**

[4.6] N non-interacting particles occupy a two non-degenerate energy levels 0 and ϵ at temperature $T = 1/(k_B \beta)$. The internal energy is given by,

(A) $3N \frac{k_B T}{2}$ (B) $N\epsilon \exp(-\beta\epsilon)$

(C) $\frac{n\epsilon}{\exp(\beta\epsilon) + 1}$ (D) $\frac{n\epsilon}{1 + \exp(-\beta\epsilon)}$ **(2 Marks)**

(Total 12 Marks)