SetTh-06 Lecture Notes
Binary Operation

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Definition 1 *To every ordered pair $\langle a, b\rangle$ of elements of a set $\mathscr{S}$ a* **binary operation** *assigns an element, denoted by $a * b$, of the set $\mathscr{S}$. For a binary operation to be a valid one it must be defined for all pairs and the $a * b$ must belong to the set and the result of binary operation must be unique.*

How a Rule may fail to define a Binary Operation

A rule that associates $a{*}b$ to an ordered pair of elements $a, b \in \mathscr{S}$ may fail to be a binary operation in following ways.

1. *$a * b$ may not be a unique element.*
2. *$a * b$ may not be defined for all pairs of elements in the set.*
3. *The result may be unique and defined but may not belong the set.*

Group as a an example of a set with a binary operation

A **group** is a pair $\langle \mathcal{G}, *\rangle$ with a binary operation $*$ defined on a set $\mathcal{G}$ such that the following axioms.

(G-1) Associative property : $a * (b * c) = (a * b) * c \qquad \forall\ a, b, c \in \mathcal{G}$

(G-2) Existence of identity : $\exists$ an element $e \in \mathcal{G}$ such that

$$e * a = a * e = a \qquad \forall\ a \in \mathcal{G}.$$

(G-3) Existence of inverse : $\forall\ a \in \mathcal{G}$ there exists an element a' such that

$$a * a' = a' * a = e$$

Remark Many books include an extra axiom called closure property. For us this property is included in the definition of binary operation.