QM Lecture Notes 6.2
General Principle of Quantum Mechanics*
Postulates of Quantum Mechanics

A.K. Kapoor
http://0space.org/users/kapoor
akkhcu@gmail.com; kapoor.proofs@gmail.com

§1 States of physical system

The state of a quantum mechanical system is represented by a vector in a complex vector space with inner product (Hilbert Space).
The null vector does not represent any state. Two non-null vectors represent the same state if and only if they are linearly dependent. Thus $|\psi_1\rangle$ and $|\psi_2\rangle$ represent the same state if there exists a complex number c such that

$$|\psi_2\rangle = c|\psi_1\rangle \tag{1}$$

A vector that represents state of physical system will be called state vector. The quantum mechanical states represented by vectors in Hilbert space are called pure states. There are other possible states which are called mixed states. These are represented by a density matrix ρ. The density matrix is an operator having properties that it is hermitian and that its eigenvalues are bewteen 0 and 1. A density matrix ρ corresponds to a pure state if and only if $\rho^2 = \rho$.

* KAPOOR //qm-lnu-06002.tex; Ver 16.x   November 13, 2016

§2 Dynamical variables

The dynamical variables of a physical system are represented by linear operators in the vector space.

A linear operator representing a dynamical variable must have real eigenvalues and their eigenvectors must form a complete set. These properties are satisfied by self adjoint operators (hermitian operators). So we demand that the dynamical variables be represented by self adjoint operators in Hilbert space. An operator representing a dynamical variable will also be called an observable.

§3 Probability and average value

If the system is in a state $|\psi\rangle$, a measurement of dynamical variable A will give one of its eigenvalues α_k with probability equal to $|\langle u_k|\psi\rangle|^2$, where $|u_k\rangle$ is normalised eigenvector of $\widehat{A}$ corresponding to eigenvalue

α_k. [1]

A result of any measurement of a dynamical variable is one of the eigenvalues of the corresponding operator. Conversely, every eigenvalue of an observable representing a dynamical variable is a possible result of a measurement of the dynamical variable.

As an example, let $|u_1\rangle, |u_2\rangle, \cdots, |u_n\rangle$ represent the eigenvectors of an observable $\hat{A}$. If the state vector of a physical system, $|u_i\rangle$, is an eigenvector of an operator $\hat{A}$ representing a dynamical variable A, a measurement of the dynamical variable gives value α with probability 1. Here α is the eigenvalue of $\hat{A}$ corresponding the eigenvector $|u_i\rangle$. Conversely, if the measurement of A gives the value α with probability 1, the state of system will be represented by a vector which will be eigenvector of the operator $\hat{A}$ corresponding to the eigenvalue α.

In general state vector $|\psi\rangle$ will not be an eigenvector of the given dynamical variable. In such a case a measurement of the variable A will results in values $\alpha_1, \alpha_2, \cdots, \alpha_n$ with probabilities $c_1|\alpha_2\rangle, c_1|\alpha_2\rangle, \cdots, c_n|\alpha_n\rangle$ where $c_1, c_2, \cdots, c_n$ are the coefficients in the expansion of the state vector $|\psi\rangle$

$$|\psi\rangle = \sum_k c_k|u_k\rangle \tag{2}$$

in terms of eigenvectors of $\hat{A}$.

Here $|\psi\rangle$ and $|u_k\rangle$ are assumed to be normalized.

$$\langle\psi|\psi\rangle = 1; \qquad \langle u_k|u_k\rangle = 1, k = 1, 2, \ldots \tag{3}$$

Flagged for adding expression for average value

[1] **Requires modification when eigenvalues of A are degenerate.**

§4 Canonical quantisation

The operators corresponding to the generalized coordinates and momenta $\{q_k, p_k\}$ of a classical system satisfy

$$\hat{q}_i\hat{q}_j - \hat{q}_j\hat{q}_i = 0 \tag{4}$$

$$\hat{p}_i\hat{p}_j - \hat{p}_j\hat{p}_i = 0 \tag{5}$$

$$\hat{q}_i\hat{p}_j - \hat{p}_j\hat{q}_i = i\hbar\delta_{ij} \tag{6}$$

The above relations are called canonical commutation relations.

§5 Time evolution

The time development of a system is governed by the Schrodinger equation

$$i\hbar\frac{d}{dt}|\psi t\rangle = \hat{H}|\psi t\rangle \tag{7}$$

where $|\psi t\rangle$ is the state vector of the system at time t and $\hat{H}$ is the operator representing the Hamiltonian of the system.

§6 Symmetrization postulate for identical particles

For a system of identical particles, the state of the system remains unchanged under exchange of a pair of particles, it should be either symmetric or antisymmetric under an exchange of all the variables of the two identical particles.[2]

KAPOOR File:qm-lnu-06002.pdf
Created: Jul 2016
Printed November 13, 2016

[2] **T&C apply**